Class X Session 2025-26
Subject - Mathematics (Basic)
Sample Question Paper - 09

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study-based questions carrying 4 marks each with sub-parts of the values of 1,1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take $\pi = 22/7$ wherever required if not stated.
11. Use of calculators is not allowed.

Section A

1. If HCF (26,169) = 13, then LCM (26,169) = **[1]**

a) 338 b) 52

c) 13 d) 26

2. The LCM and HCF of two rational numbers are equal, then the numbers must be **[1]**

a) equal b) co-prime

c) composite d) prime

3. If **p** is a root of the quadratic equation x^2 - (p + q) x + k = 0, then the value of **k** is **[1]**

a) pq b) p + q

c) q d) p

4. The value of k, for which the system of equations kx - 3y + 6 = 0, 4x - 6y + 15 = 0 represent parallel lines, is **[1]**

________.

a) 2 b) 1

c) 3 d) 4

5. A quadratic equation $ax^2 + bx + c = 0$ has real and equal roots, if **[1]**

a) $b^2 - 4ac > 0$ b) $b^2 - 4ac = 0$

c) $b^2 - 4ac < 0$ d) $b^2 - 4ac \neq 0$

6. If A (-1, 0), B(5, -2) and C(8, 2) are the vertices of a $\triangle ABC$ then its centroid is **[1]**

a) (0, 6) b) (6, 0)

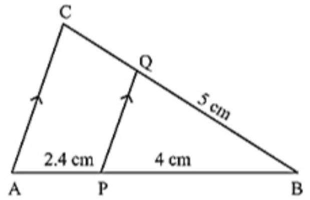
c) (12, 0) d) (4, 0)

7. $\triangle PQR \sim \triangle XYZ$ and the perimeters of $\triangle PQR$ and $\triangle XYZ$ are 30 cm and 18 cm respectively. If QR = 9 cm, then, YZ is equal to **[1]**

a) 5.4 cm. b) 4.5 cm.

c) 9.5 cm. d) 12.5 cm.

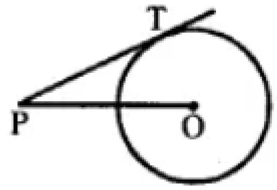
8. In the given figure, PQ || AC. If BP = 4 cm, AP = 2.4 cm and BQ = 5 cm, then length of BC is: **[1]**



a) 8 cm b) $\frac{25}{3}$ cm

c) 0.3 cm d) 3 cm

9. In the given figure, point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 24 cm. Then, the radius of the circle is **[1]**



a) 10 cm b) 12 cm

c) 13 cm d) 15 cm

10. If $x = a\cos\theta$ and $y = b\sin\theta$, then the value of $b^2x^2 + a^2y^2$ is: **[1]**

a) ab b) $a^2 + b^2$

c) a^4b^4 d) a^2b^2

11. The angles of depression of two ships from the top of a lighthouse are 45° and 30° towards east. If the ships are 100 m apart, the height of the lighthouse is **[1]**

a) $50(\sqrt{3} - 1)$ m b) $\frac{50}{\sqrt{3}+1}$ m

c) $50(\sqrt{3} + 1)$ m d) $\frac{50}{\sqrt{3}-1}$ m

12. [$\frac{3}{4}$ $\tan^2 30^o$ - $\sec^2 45^o$ + $\sin^2 60^o$] is equal to **[1]**

a) $\frac{-3}{2}$

b) $\frac{5}{6}$

c) -1

d) $\frac{1}{6}$

13. The area of the circle that can be inscribed in a square of side 10 cm is **[1]**

a) $20\pi\ sq.cm$

b) 25 sq.cm

c) $25\pi\ sq.cm$

d) $10\pi\ sq.cm$

14. If the area of a sector of a circle is $\frac{5}{18}$ of the area of the circle, then the sector angle is equal to **[1]**

a) 60°

b) 90°

c) 100°

d) 120°

15. Cards bearing numbers 3 to 20 are placed in a bag and mixed thoroughly. A card is taken out of the bag at random. What is the probability that the number on the card taken out is an even number? **[1]**

a) $\frac{9}{17}$

b) $\frac{5}{9}$

c) $\frac{1}{2}$

d) $\frac{7}{18}$

16. Consider the following distribution : **[1]**

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

the frequency of the class 30-40 is

a) 48

b) 4

c) 3

d) 51

17. An icecream cone has hemispherical top. If the height of the cone is 9 cm and base radius is 2.5 cm, then the volume of icecream is **[1]**

a) 91.67 cm^3

b) 96.67 cm^3

c) 91.76 cm^3

d) 90.67 cm^3

18. If mode of a series exceeds its mean by 12, then mode exceeds the median by: **[1]**

a) 8

b) 6

c) 10

d) 4

19. **Assertion (A):** Image of point (–5, 0) is (5, 0). **[1]**

Reason (R): Image of point (a, 0) is (–a, 0).

a) Both A and R are true and R is the correct

b) Both A and R are true but R is not the explanation of A. correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** H.C.F. of 12 and 77 is 1. **[1]**

Reason (R): L.C.M. of two coprime numbers is equal to their product.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Use elimination method to find all possible solutions of the following pair of linear equations: **[2]**

2x + 3y = 8 ...(1)

4x + 6y = 7 ...(2)

22. If D and E are points on sides AB and AC respectively of a ΔABC such that $DE \| BC$ and BD = CE. Prove that Δ ABC is isosceles. **[2]**

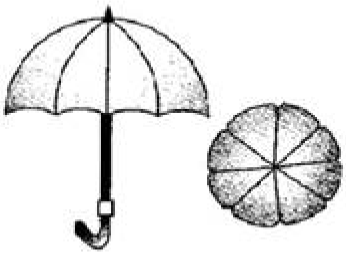
OR

If $\triangle \mathrm{ABC} \cong \triangle \mathrm{DEF}$, $\angle A = 47^\circ$, $\angle E = 83^\circ$, then find $\angle \mathrm{C}$.

23. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. **[2]**

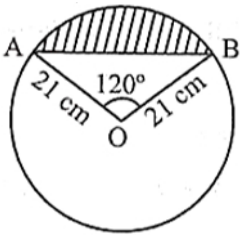
24. Find A and B if sin (A + 2B) = $\frac{\sqrt{3}}{2}$ and cos (A + 4B) = 0, where A and B are acute angles. **[2]**

25. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, Find the area between the two consecutive ribs of the umbrella. **[2]**



OR

Find the area of the segment shown in Fig., if radius of the circle is 21 cm and ∠AOB = 120^o (Use π = $\frac{22}{7}$)



Section C

26. Show that $5 - \sqrt{3}$ is irrational. **[3]**

27. Find the zeroes of the polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ by factorisation method and verify the relationship between the zeroes and coefficient of the polynomial. **[3]**

28. Champa went to a **Sale** to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, **The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased**. Help her friends to find how many pants and skirts Champa bought. **[3]**

OR

A two-digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

29. Prove that the angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre **[3]**

30. Prove the trigonometric identity: $\frac{\cot^2\theta(\sec\theta-1)}{(1+\sin\theta)} + \frac{\sec^2\theta(\sin\theta-1)}{(1+\sec\theta)} = 0$ **[3]**

OR

Prove that $\sec\theta(1 - \sin\theta)(\sec\theta + \tan\theta) = 1$

31. In a bag there are 44 identical cards with figure of circle or square on them. There are 24 circles, of which 9 are blue and rest are green and 20 squares of which 11 are blue and rest are green. One card is drawn from the bag at random. Find the probability that it has the figure of **[3]**

i. square
ii. green colour,
iii. blue circle and
iv. green square.

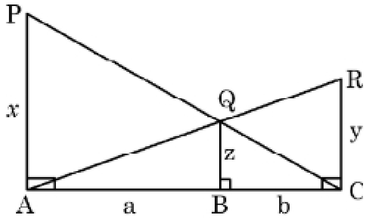
Section D

32. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what was its first average speed? **[5]**

OR

At t minutes past 2 p.m, the time needed by the minute hand of a clock to show 3 p.m. was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t.

33. PA, QB and RC are each perpendicular to AC. If AP = x, QB = z, RC = y, AB = a and BC = b, then prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$. **[5]**



34. A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is equal to the radius of the cone. **[5]**

OR

A spherical glass vessel has a cylindrical neck 8 cm long and 1 cm in radius. The radius of the spherical part is 9 cm. Find the amount of water (in litres) it can hold, when filled completely.

35. The following table gives the distribution of the life time of 400 neon lamps: **[5]**

Lite time (in hours)	**Number of lamps**
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74

4000-4500	62
4500-5000	48

Find the median life time of a lamp.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Deepa has to buy a scooty. She can buy scooty either making cashdown payment of ₹ 25,000 or by making 15 monthly instalments as below.

Ist month - ₹ 3425, IInd month - ₹ 3225, IIIrd month - ₹ 3025, IVth month - ₹ 2825 and so on

i. Find the amount of 6th instalment. (1)
ii. Total amount paid in 15 instalments. (1)
iii. Deepa paid 10th and 11th instalment together find the amount paid that month. (2)

OR

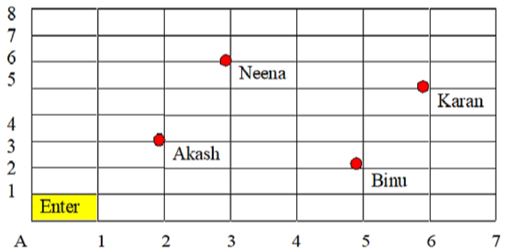
If Deepa pays ₹2625 then find the number of instalment. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Karan went to the Lab near to his home for COVID 19 test along with his family members.

The seats in the waiting area were as per the norms of distancing during this pandemic (as shown in the figure).

His family member took their seats surrounded by red circular area.



i. What is the distance between Neena and Karan? (1)
ii. What are the coordinates of seat of Akash? (1)
iii. What will be the coordinates of a point exactly between Akash and Binu where a person can be? (2)

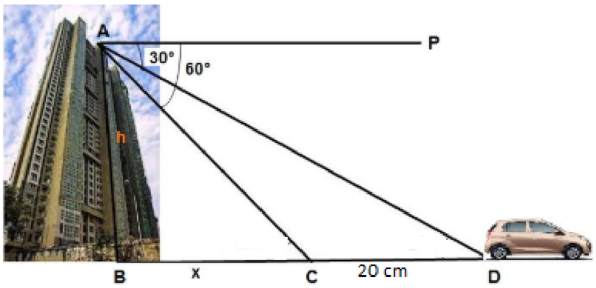
OR

Find distance between Binu and Karan. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Vijay lives in a flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^{o}. After accelerating 20 m from point C, Vijay stops at point D to buy ice cream and the angle of depression changed to

30^o.



i. Find the value of x. (1)

ii. Find the height of the building AB. (1)

iii. Find the distance between top of the building and a car at position D? (2)

OR

Find the distance between top of the building and a car at position C? (2)

Solution

Section A

1. **(a)** 338

   Explanation:

   HCF (26, 169) = 13

   We have to find the value for LCM (26, 169)

   We know that the product of numbers is equal to the product of their HCF and LCM.

   Therefore,

   13(LCM) = 26(169)

   $\text{LCM} = \frac{26(169)}{13}$

   LCM = 338

2. **(a)** equal

   Explanation:

   If we assume that a and b are equal and consider a = b = k

   Then,

   HCF (a, b)= k

   LCM (a, b) = k

3. **(a)** pq

   Explanation:

   Let the roots of given quadratic equation be α and β.

   On comparing equation $x^2 - (p - q)x + k = 0$

   with $ax^2 + bx + c = 0$, we have

   a = 1, b = -(p + q), c = k

   We know that

   $\Rightarrow \alpha + \beta = \frac{-b}{a}$

   Put the value a and b

   $\Rightarrow \alpha + \beta = \frac{p+q}{1}$

   $\Rightarrow \alpha + \beta$ = p + q ...(i)

   Given α = p

   Put the value of α in equation (i),

   $\Rightarrow$ p + β = p + q

   $\Rightarrow \beta$ = q

   But we know that

   $\alpha \cdot \beta = \frac{c}{a}$

   Put the values

   p.q. $= \frac{k}{1}$

   Then, k = pq.

4. **(a)** 2

   Explanation:

   It is given that, kx - 3y + 6 = 0 and 4x - 6y + 15 = 0 are two parallel lines.

   i.e., the given lines has no solution

   $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

   $\Rightarrow \frac{k}{4} = \frac{-3}{-6} \neq \frac{k}{4} = \frac{3}{6} \Rightarrow k = 2$

5. **(b)** $b^2 - 4ac = 0$

   Explanation:

A quadratic equation $ax^2 + bx + c = 0$ has real and equal roots, if $b^2 - 4ac = 0$.

6.

(d) (4, 0)

Explanation:

Centriod is G $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = G\left(\frac{-1+5+8}{3}, \frac{0-2+2}{3}\right) = (4, 0)$

7. **(a)** 5.4 cm.

Explanation:

Given: $\triangle PQR \sim \triangle XYZ$

$\therefore \frac{\text{Perimeter of } \Delta PQR}{\text{Perimeter of } \Delta XYZ} = \frac{QR}{YZ}$

$\Rightarrow \frac{30}{18} = \frac{9}{YZ}$

$\Rightarrow$YZ = 5.4 cm

8. **(a)** 8 cm

Explanation:

As PQ || AC by using proportionality theorem

$\Rightarrow \frac{BP}{PA} = \frac{BQ}{QC}$

$\Rightarrow \frac{4}{2.4} = \frac{5}{QC}$

$\Rightarrow QC = \frac{5 \times 2.4}{4}$ = 5 × 0.6

$\Rightarrow$ QC = 3 cm

$\therefore$ BC = BQ + QC

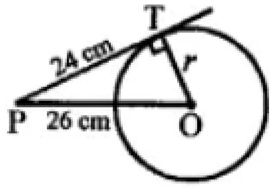
= 5 + 3

= 8 cm

9. **(a)** 10 cm

Explanation:

In the given figure, point P is 26 cm away from the centre O of the circle.



Length of tangent PT = 24 cm

Let radius = r

In right $\triangle$OPT,

$OP^2 = PT^2 + OT^2$

$\Rightarrow 26^2 = 24^2 + r^2$

$\Rightarrow r^2 = 26^2 - 24^2 = 676 - 576 = 100 = (10)^2$

r = 10

Radius = 10 cm

10.

(d) a^2b^2

Explanation:

Given: $x = a\cos\theta$ and $y = b\sin\theta$

$\therefore b^2x^2 + a^2y^2$

$= b^2(a\cos\theta)^2 + a^2(b\sin\theta)^2$

$= b^2a^2\cos^2\theta + a^2b^2\sin^2\theta$

$\Rightarrow b^2x^2 + a^2y^2$

$= a^2b^2(\cos^2\theta + \sin^2\theta)$

$= a^2b^2$

$[\because \sin^2\theta + \cos^2\theta = 1]$

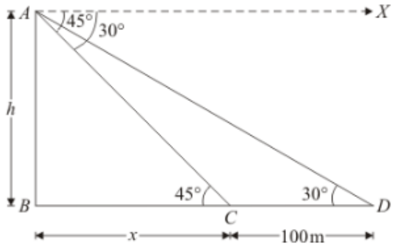
11.

(c) $50(\sqrt{3}+1)$ m

Explanation:

Let AB = h be the lighthouse.

The given situation can be represented as,



It is clear that $\angle C = 45^o$ and $\angle D = 30^o$

Again, let BC = x and CD = 100 m is given.

Here, we have to find the height of lighthouse.

So we use trigonometric ratios.

In a triangle ABC,

$\Rightarrow \tan C = \frac{AB}{BC}$

$\Rightarrow \tan 45^\circ = \frac{h}{x}$

$\Rightarrow 1 = \frac{h}{x}$

$\Rightarrow h = x$

Again in a triangle *ABD*,

$\Rightarrow \tan D = \frac{AB}{BC+CD}$

$\Rightarrow \tan 30^\circ = \frac{h}{x+100}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+100}$

$\Rightarrow \sqrt{3}h = x + 100$

Put x = h

$\Rightarrow \sqrt{3}\, h = h + 100$

$\Rightarrow h(\sqrt{3} - 1) = 100$

$\Rightarrow h = \frac{100}{\sqrt{3}-1}$

$\Rightarrow h = \frac{100}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow h = 50(\sqrt{3} + 1)$

12.

(c) -1

Explanation:

$\frac{3}{4}\tan^2 30^o - \sec^2 45^o + \sin^2 60^o$

$= \frac{3}{4}(\frac{1}{\sqrt{3}})^2 = (\sqrt{2})^2 + (\frac{\sqrt{3}}{2})^2$

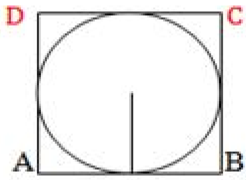
$= \frac{1}{4} - 2 + \frac{3}{4}$

$= \frac{1-8+3}{4} = \frac{4-8}{4} = -1$

13.

(c) $25\pi\ sq.\ cm$

Explanation:



Since if a circle inscribed a square,
then the radius of the circle is half of the side of the square.
$\therefore$ Radius = $\frac{10}{2}$ = 5 cm
$\therefore$ Area of the circle = πr^2 = $\pi(5)^2$ = 25π sq. cm

14.
(c) 100°
Explanation:
We have given that area of the sector is $\frac{5}{18}$ of the area of the circle.
Therefore, area of the sector $= \frac{5}{18} \times$ area of the circle
$\Rightarrow \frac{\theta}{360} \times \pi r^2 = \frac{5}{18} \times \pi r^2$
Now we will simplify the equation as below,
$\Rightarrow \frac{\theta}{360} = \frac{5}{18}$
$\therefore \theta = \frac{5}{18} \times 360$
$\therefore \theta = 100$
Therefore, sector angle is 100°.

15.
(c) $\frac{1}{2}$
Explanation:
Total number of card = 18
n(s) = 18
A = Even numbers from 3 to 20
(A) = 4, 6, 8, 10, 12, 14, 16, 18, 20
n(A) = 9
Required probability P(A) = $\frac{n(A)}{n(S)} = \frac{9}{18}$
P(A) = $\frac{1}{2}$

16.
(c) 3
Explanation:

Marks Obtained	Number of students	f
0-10	(63-58)=5	5
10-20	(58-55)=3	3
20-30	(55-51)=4	4
30-40	(51-48)=3	3
40-50	(48-42)=6	6
50...	42=42	42

Hence, frequency in the class interval 30 - 40 is 3.

17. **(a)** 91.67 cm^3
Explanation:

Height of ice-cream cone is 9 cm and radius of the hemispherical top is 2.5 cm.

Now, Volume of ice-cream cone = Volume of cone + volume of Hemispherical top

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2(h + 2r)$

$= \frac{1}{3} \times \frac{22}{7} \times 2.5 \times 2.5(9 + 5)$

$= \frac{1}{3} \times \frac{22}{7} \times 2.5 \times 2.5 \times 14$

$= 91.67 \text{ cm}^3$

18. **(a)** 8

Explanation:

Mode of a series = Its mean + 12

Mean = mode - 12

Also we know that

Mode = 3 median - 2 Mean

$\Rightarrow$ Mode = 3 median - 2(mode - 12)

$\Rightarrow$ Mode = 3 median - 2 mode + 24

$\Rightarrow$ Mode + 2 mode - 3 median = 24

$\Rightarrow$ 3 mode - 3 median = 24

$\Rightarrow$ 3(mode - median) = 24

$\Rightarrow$ Mode - median $= \frac{24}{3} = 8$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Image of points of type (h, 0) is (–h, 0) only.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Yes 12 and 17 are coprime numbers and H.C.F. of coprimes is always 1.

Section B

21. Step 1: Multiply equation (i) by 2 and equation (ii) by 1 to make the coefficients of x equal. Then we get the equations as :

4x + 6y = 16 ...(iii)

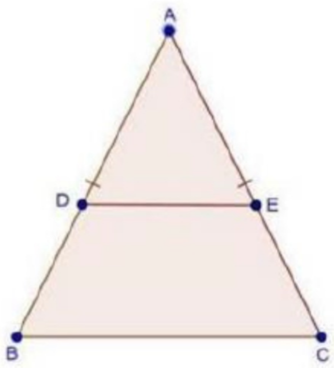
4x + 6y = 7 ...(iv)

Step 2: Subtracting equation (iv) from equation (iii),we get

(4x - 4x) + (6y - 6y) = 16 - 7

i.e., 0 = 9, which is a false statement. Therefore, the given pair of equations has no solution.

22. We have, DE || BC



Therefore, by BPT, $\frac{AD}{BD} = \frac{AE}{EC} \Rightarrow AD = AE$

Adding DB on both sides

$\Rightarrow$ AD + DB = AE + DB

$\Rightarrow$ AD + DB = AE + EC   [$\because$ BD = CE]

$\Rightarrow$ AB = AC

∴ Δ ABC is isosceles triangle.

OR

Given: △ABC ≅ △DEF

⇒∠A = ∠D, ∠B = ∠E, ∠C = ∠F

Also ∠A = 47^o, ∠E = 83^o =∠B

By angle sum property for triangles,

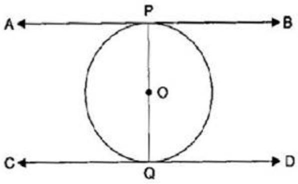
∠A + ∠B + ∠C = 180^o

47^o + 83^o + ∠C = 180^o

130^o + ∠C = 180^o

∠C = 180^o - 130^o

∠C = 50^o

23. 

Given: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.

To Prove: AB || CD

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

∴ ∠OPA = 90^o .... (i)

[The tangent at any point of a circle is ⊥ to the radius through the point of contact]

∵ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

∴ ∠OQD = 90^o .... (ii)

[The tangent at any point of a circle is ⊥ to the radius through the point of contact]

From eq. (i) and (ii), ∠OPA = ∠OQD

But these form a pair of equal alternate angles also,

∴ AB || CD

24. A + 2B = 60^o and A + 4B = 90^o

Solving to get B = 15^o and A = 30^o

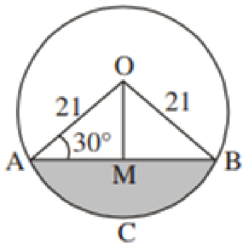
25. Here, r = 45 cm and $\theta = \frac{360^\circ}{8} = 45^\circ$

Area between two consecutive ribs of the umbrella = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45 \times 45 \quad = \frac{22275}{28}$ cm ^{2}

OR

Draw OM ⊥ AB



∠OAB = ∠OBA = 30^o

sin 30^o = $\frac{1}{2} = \frac{OM}{21} \Rightarrow$ OM = $\frac{21}{2}$

cos 30^o = $\frac{\sqrt{3}}{2} = \frac{AM}{21} \Rightarrow$ AM = $\frac{21}{2}\sqrt{3}$

Area of △OAB = $\frac{1}{2} \times AB \times OM = \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$

= $\frac{441}{4}\sqrt{3}$ cm^2

∴ Area of shaded region = Area (sector OACB) - Area (△OAB)

$= \frac{22}{7} \times 21 \times 21 \times \frac{120}{360} - \frac{441}{4}\sqrt{3}$

$= \left(462 - 441\frac{\sqrt{3}}{4}\right)$ cm^2 or 271.3 cm^2 (approx.)

Section C

26. Let us assume, to the contrary, that $5 - \sqrt{3}$ is rational.

That is, we can find coprime numbers a and b (b ≠ 0) such that $5 - \sqrt{3} = \frac{a}{b}$

Therefore, $5 - \frac{a}{b} = \sqrt{3}$

Rearranging this equation, we get $\sqrt{3} = 5 - \frac{a}{b} = \frac{5b-a}{b}$

Since a and b are integers, we get $5 - \frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational

This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.

So, we conclude that $5 - \sqrt{3}$ is irrational.

27. $7y^2 - \frac{11}{3}y - \frac{2}{3}$

$= \frac{1}{3}(21y^2 - 11y - 2)$

$= \frac{1}{3}(21y^2 - 14y + 3y - 2)$

$= \frac{1}{3}[7y(3y - 2) + 1(3y - 2)]$

$= \frac{1}{3}(3y - 2)(7y + 1)$

$\Rightarrow y = \frac{2}{3}, \frac{-1}{7}$ are zeroes of the polynomial.

If Given polynimoal is $7y^2 - \frac{11}{3}y - \frac{2}{3}$

Then a = 7 , b = $-\frac{11}{3}$ and c = $-\frac{2}{3}$

Sum of zeroes = $\frac{2}{3} + \frac{-1}{7} = \frac{14-3}{21} = \frac{11}{21}$ .... (i)

Also, $\frac{-b}{a} = \frac{-\left(\frac{-11}{3}\right)}{7} = \frac{11}{21}$ .... (ii)

From (i) and (ii)

Sum of zeroes = $\frac{-b}{a}$

Now, product of zeroes = $\frac{2}{3} \times \frac{-1}{7} = \frac{-2}{21}$ .... (iii)

Also, $\frac{c}{a} = \frac{\frac{-2}{3}}{7} = \frac{-2}{21}$ .... (iv)

From (iii) and (iv)

Product of zeroes = $\frac{c}{a}$

28. Let us denote the number of pants by x and the number of skirts by y.

Then the equations formed are:

y = 2x - 2 .... (i)

y = 4x - 4...........(ii)

From (i)

When x = 2, then y = 2

When x = 1, then y = 0

x	2	1
y	2	0

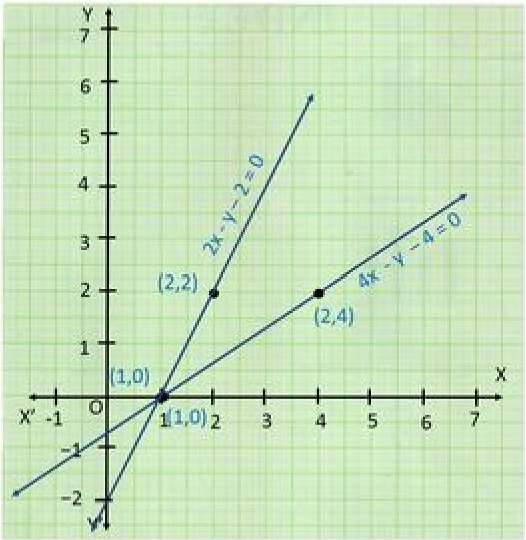
From (ii)

When x = 2, then y = 4

When x = 1, then y = 0

x	2	1
y	4	0

The graphs of two equations of line is shown below.



From the graph, the lines intersect at point (1, 0)

Thus, the value of x = 1 and y = 0

Hence, the number of pants she purchased are 2 and the number of skirts she purchased are 0.

OR

Let us suppose that the digit at unit place be x

Suppose the digit at tens place be y.

Thus, the number is 10y + x.

According to question it is given that the number is 4 times the sum of the two digits.

Therefore, we have

$10y + x = 4(x + y)$

$\Rightarrow 10y + x = 4x + 4y$

$\Rightarrow 4x + 4y - 10y - x = 0$

$\Rightarrow 3x - 6y = 0$

$\Rightarrow 3(x - 2y) = 0$

$\Rightarrow x - 2y = 0$

After interchanging the digits, the number becomes 10x + y.

Again according to question If 18 is added to the number, the digits are reversed.

Thus, we have

$(10y + x) + 18 = 10x + y$

$\Rightarrow 10x + y - 10y - x = 18$

$\Rightarrow 9x - 9y = 18$

$\Rightarrow 9(x - y) = 18$

$\Rightarrow x - y = \frac{18}{9}$

$\Rightarrow x - y = 2$

Therefore, we have the following systems of equations

$x - 2y = 0$ ....(1)

$x - y = 2$...............(2)

Here x and y are unknowns. Now let us solve the above systems of equations for x and y.

Subtracting the equation (1) from the (2), we get

$(x - y) - (x - 2y) = 2 - 0$

$\Rightarrow x - y - x + 2y = 2$

$\Rightarrow y = 2$

Now, substitute the value of y in equation (1), we get

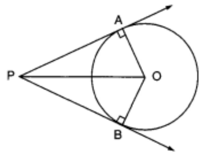
$x - 2 \times 2 = 0$

$\Rightarrow x - 4 = 0$

$\Rightarrow x = 4$

Therefore the number is $10 \times 2 + 4 = 24$

Thus the number is 24

29. 

Let PA and PB be two tangents drawn from an external point P to a circle with centre O.

We have to prove that angles $\angle AOB$ and $\angle APB$ are supplementary i.e.

$\angle AOB + \angle APB$ = 180°.

In right triangles OAP and OBP, we have

PA = PB [Tangents drawn from an external point are equal]

OA = OB [Each equal to radius]

and, OP = OP [Common]

So, by SSS-criterion of congruence, we obtain

$\Delta OAP \cong \Delta OBP$

$\angle OPA = \angle OPB$

$\angle AOP = \angle BOP$

$\angle APB = 2\angle OPA$ ......(1)

$\angle AOB = 2\angle AOP$ ......(2)

$\angle AOP = 90^\circ - \angle OPA$ [$\because \Delta OAP$ is right triangle]

$2\angle AOP = 180^\circ - 2\angle OPA$

$\angle AOB = 180^\circ - \angle APB$ [Using(1) and (2)]

$\angle AOB + \angle APB = 180^\circ$

30. LHS = $\frac{\cot^2\theta(\sec\theta-1)}{(1+\sin\theta)} + \frac{\sec^2\theta(\sin\theta-1)}{(1+\sec\theta)}$

$= \frac{\cot^2\theta(\sec\theta-1)(1+\sec\theta)+\sec^2\theta(\sin\theta-1)(1+\sin\theta)}{(1+\sin\theta)(1+\sec\theta)}$

$= \frac{\cot^2\theta(\sec^2\theta-1)+\sec^2\theta(\sin^2\theta-1)}{(1+\sin\theta)(1+\sec\theta)}$

$= \frac{\cot^2\theta\tan^2\theta+\sec^2\theta(-\cos^2\theta)}{(1+\sin\theta)(1+\sec\theta)}$

$= \frac{\cot^2\theta\tan^2\theta-\sec^2\theta\cos^2\theta}{(1+\sin\theta)(1+\sec\theta)}$

$= \frac{\cot^2\theta\times\frac{1}{\cot^2\theta}-\sec^2\theta\times\frac{1}{\sec^2\theta}}{(1+\sin\theta)(1+\sec\theta)}$

$= \frac{1-1}{(1+\sin\theta)(1+\sec\theta)}$

= 0

= RHS

OR

LHS = secθ(1 - sinθ)(secθ + tanθ)

$= \left[\sec\theta - \frac{\sin\theta}{\cos\theta}\right] \times (\sec\theta + \tan\theta)$

$= (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$

= $\sec^2\theta$ - $\tan^2\theta$ = 1 = RHS

31. Number of identical cards = 44

Out of 44 cards, one card can be drawn in 44 ways.

$\therefore$ Total number of elementary events = 44

Number of circles = 24

Number of blue circles = 9

$\therefore$ Number of green circles = 24 - 9 = 15

Number of squares = 20

Number of blue squares = 11

$\therefore$ Number of green squares = 20 - 11 = 9

i. Number of square = 20

$\therefore$ Favourable number of elementary events = 20

Hence, required probability = $\frac{20}{44} = \frac{5}{11}$

ii. Number of green figures = Number of green circles + Number of green square

=15 + 9 = 24

∴ Favourable number of elementary events = 24

Hence, required probability = $\frac{24}{44}=\frac{6}{11}$

iii. Number of blue circles = 9

∴ Favourable number of elementary events = 9

Hence, required probability = $\frac{9}{44}$

iv. Number of green squares = 9

∴ Favourable number of elementary events = 9

Hence, required probability = $\frac{9}{44}$.

Section D

32. Let the speed of the train be x km/hr for first 54 km and for next 63 km, speed is (x + 6) km/hr.

According to the question

$\frac{54}{x}+\frac{63}{x+6}=3$

$\frac{54(x+6)+63x}{x(x+6)}=3$

or, 54x + 324 + 63x = 3x(x + 6)

or, $117x+324=3x^2+18x$

or, $3x^2-99x-324=0$

or, $x^2-33x-108=0$

or, $x^2-36x+3x-108=0$

or, x(x - 36) + 3(x - 36) = 0

(x - 36)(x + 3) = 0

x = 36

x = - 3 rejected.

(as speed is never negative)

Hence First speed of train = 36 km/h

OR

Total time taken by minute hand from 2 p.m. to 3 p.m. is 60 min.

According to question,

$t+\left(\frac{t^2}{4}-3\right)=60$

⇒ $4t + t^2 – 12 = 240$

⇒ $t^2 + 4t – 252 = 0$

⇒ $t^2 + 18t – 14t – 252 = 0$

⇒ t(t + 18) – 14(t +18) = 0

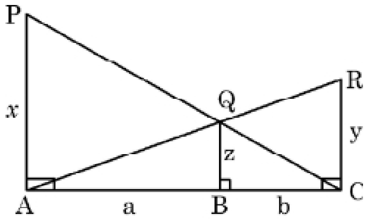
⇒ (t + 18) (t – 14) = 0

⇒ t + 18 = 0 or t – 14 = 0

⇒ t = –18 or t = 14 min.

As time can't be negative.

Therefore, t = 14 min.

33. 

In the given figure we have $PA\perp AC$ and $QB\perp AC$

$\Rightarrow QB\| PA$

In $\triangle PAC$ and $\triangle QBC$, we have

$\angle QCB=\angle PCA$ (Common)

$\angle QBC=\angle PAC$ (both are 90^o).

So by AA similarity rule , $\triangle QBC\sim\triangle PAC$

$\therefore \frac{QB}{PA}=\frac{BC}{AC}$

$\Rightarrow \frac{z}{x}=\frac{b}{a+b}$ ...(i) [by the property of similar triangles]

In $\triangle RAC$, $QB \| RC$.

So, $\triangle QBA \sim \triangle RCA$.

$\therefore \frac{QB}{RC} = \frac{AB}{AC}$

$\Rightarrow \frac{z}{y} = \frac{a}{a+b}$ ...(ii) [by the property of similar triangles]

Form (i) and (ii), we obtain

$\frac{z}{x} + \frac{z}{y} = \left(\frac{b}{a+b} + \frac{a}{a+b}\right) = 1$

$\Rightarrow \frac{z}{x} + \frac{z}{y} = 1$

$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

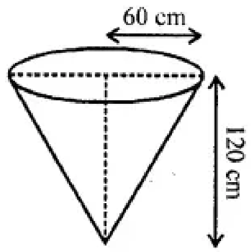
or $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.

Hence proved.

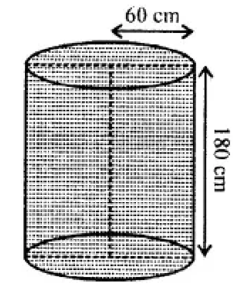
34. i. Whenever we placed a solid right circular cone in a right circular cylinder ,cylinder with full of water, then volume of a solid right circular cone is equal to the volume of water filled from the cylinder.

ii. Total volume of water in a cylinder is equal to the volume of the cylinder.

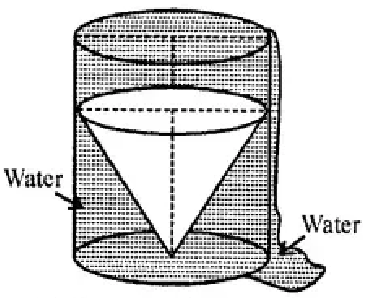
iii. Volume of water left in the cylinder is = Volume of the right circular cylinder - Volume of a right circular cone.



(Right circular cone)



(Right circular cylinder)



(Cylinder contained a cone)

Now, given that

Height of a right circular cone = 120cm

Radius of a right circular cone = 60cm

$\therefore$ The volume of a right circular cone = $\left(\frac{1}{3}\right) \pi r^2 \times h$

$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 60 \times 60 \times 120$

$= \left(\frac{22}{7}\right) \times 20 \times 60 \times 120$

= 14000 π cm^3

$\therefore$ Volume of a right circular cone = Volume of water spilled from the cylinder =144000π cm^3 [from point (i)]

Given that, the height of a right circular cylinder =180cm

and radius of a right circular cylinder = Radius of a right circular cone = 60 cm

$\therefore$ Volume of a right circular cylinder $= \pi r^2 \times$ h

$= \pi \times 60 \times 60 \times 180 = 648000\pi$ cm^3 So, volume of a right circular cylinder = Total volume of water in a cylinder = 648000π cm^3 [from point (ii)]

From point (iii),

Volume of water left in the cylinder = Total volume of water in a cylinder - Volume of water failed from the cylinder when solid cone is placed in it

$= 648000\pi - 144000\pi$

$= 504000\pi = 504000 \times \left(\frac{22}{7}\right) = 1584000$ cm^3

$= \left(\frac{1584000}{(10)6}\right) m^3 = 1.584 m^3$

Hence, the required volume of water left in the cylinder is 1.584 m^3

OR

The volume of the spherical vessel is
calculated by the given formula

V = $\frac{4}{3}\pi \times r^3$

Now,

V = $\frac{4}{3} \times \frac{22}{7} \times 9 \times 9 \times 9$

V = 3,054.85 cm^3

The volume of the cylinder neck is calculated by the given formula.

V = $\pi \times R^2 \times h$

Now,

V = $\frac{22}{7} \times 1 \times 1 \times 8$

V = 25.14 cm^3

The total volume of the vessel is equal to the volume of the spherical shell and the volume of its cylindrical neck.

3054.85 + 25.14 = 3, 080 cm^3

The total volume of the vessel is 3,080 cm^3.

As we know,

1 L = 1000 cm^3

$\frac{3080}{1000}$ = 3.080 L

Thus, the amount of water (in litres) it can hold is 3.080 L.

35.

Life time	Number of lamps (f_i)	Cumulative frequency
1500-2000	14	14
2000-2500	56	14 + 56 = 70
2500-300	60	70 + 60 = 130
3000-3500	86	130 + 86 = 216
3500-4000	74	216 + 74 = 290
4000-4500	62	290 + 62 = 352
4500-5000	48	352 + 48 = 400
	400	

N = 400

Now we may observe that cumulative frequency just greater than $\frac{n}{2}$ (ie., $\frac{400}{2}$ = 200) is 216

Median class = 3000 - 3500

Median = l + $\left(\frac{\frac{n}{2} - cf}{f}\right) \times$ h

Here,

l = Lower limit of median class

F = Cumulative frequency of class prior to median class.

f = Frequency of median class.

h = Class size.

Lower limit (l) of median class = 3000

Frequency (f) of median class 86

Cumulative frequency (cf) of class preceding median class = 130

Class size (h) = 500

Median = 3000 + $\left(\frac{200-130}{86}\right) \times$ 500

= 3000 + $\frac{70 \times 500}{86}$

= 3406.98

Section E

36. i. 1st installment = ₹ 3425

2nd installment = ₹ 3225

3rd installment = ₹ 3025

and so on

Now, 3425, 3225, 3025, ... are in AP, with

a = 3425, d = 3225 - 3425 = -200

Now 6th installment = a_n = a + 5d = 3425 + 5(-200) = ₹ 2425

ii. Total amount paid = $\frac{15}{2}$(2a + 14d)

= $\frac{15}{2}$[2(3425) + 14(-200)] = $\frac{15}{2}$(6850 - 2800)

= $\frac{15}{2}$(4050) = ₹ 30375

iii. $a_n = a + (n - 1)d$

$\Rightarrow a_{10} = 3425 + 9 \times (-200)$ = 1625

$\Rightarrow a_{11} = 3425 + 10 \times (-200) = 1425$

$a_{10} + a_{11} = 1625 + 1425 = 3050$

OR

$a_n = a + (n - 1)d$ given $a_n = 2625$

$2625 = 3425 + (n - 1) \times - 200$

$\Rightarrow - 800 = (n - 1) \times - 200$

$\Rightarrow 4 = n - 1$

$\Rightarrow n = 5$

So, in 5th installment, she pays ₹ 2625.

37. i. Position of Neena = (3, 6)

Position of Karan = (6, 5)

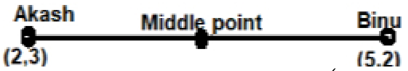
Distance between Neena and Karan = $\sqrt{(6-3)^2 + (5-6)^2}$

= $\sqrt{9 + (-1)^2}$

= $\sqrt{10}$

ii. Co-ordinate of seat of Akash = 2, 3

iii.



Co-ordinate of middle point = $\left(\frac{2+5}{2}, \frac{3+2}{2}\right)$

= 3.5, 2.5

OR

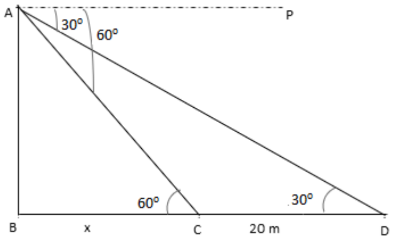
Binu = (5, 5); Karan = (6, 5)

Distance = $\sqrt{(6-5)^2 + (5-2)^2}$

$= \sqrt{1+9}$

$= \sqrt{10}$

38. i. The above figure can be redrawn as shown below:



From the figure,

let AB = h and BC = x

In $\triangle$ABC,

tan 60 = $\frac{AB}{BC} = \frac{h}{x}$

$\sqrt{3} = \frac{h}{x}$

h = $\sqrt{3}$x ...(i)

In $\triangle$ABD,

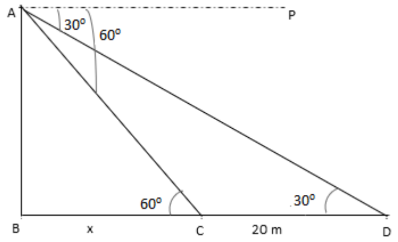
tan 30 = $\frac{AB}{BD} = \frac{h}{x+20}$

$\frac{1}{\sqrt{3}}$ = $\frac{\sqrt{3}x}{x+20}$ [using (i)]

x + 20 = 3x

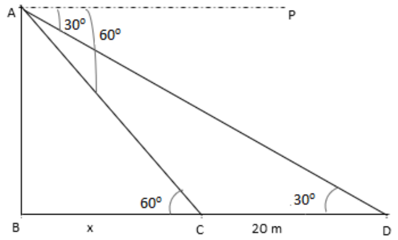
x = 10 m

ii. The above figure can be redrawn as shown below:



Height of the building, h = $\sqrt{3}$x = 10$\sqrt{3}$ = 17.32 m

iii. The above figure can be redrawn as shown below:



Distance from top of the building to point D.

In $\triangle$ABD

sin 30^0 = $\frac{AB}{AD}$

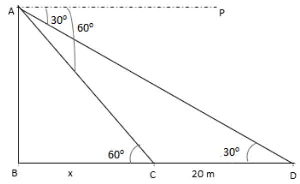
$\Rightarrow AD = \frac{AB}{\sin 30^0}$

$\Rightarrow AD = \frac{10\sqrt{3}}{\frac{1}{2}}$

$\Rightarrow$ AD = $20\sqrt{3}m$

OR

The above figure can be redrawn as shown below:



Distance from top of the building to point C is

In $\triangle$ABC

sin 60^0 = $\frac{AB}{AC}$

$\Rightarrow AC = \frac{AB}{\sin 60^0}$

$\Rightarrow AC = \frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AD = 20 m